

MENSURATION

Definition

- Mensuration** : It is a branch of mathematics which deals with the lengths of lines, areas of surfaces and volumes of solids.
- Plane Mensuration** : It deals with the sides, perimeters and areas of plane figures of different shapes.
- Solid Mensuration** : It deals with the areas and volumes of solid objects.

Important Formulae

Right Angled Triangle :

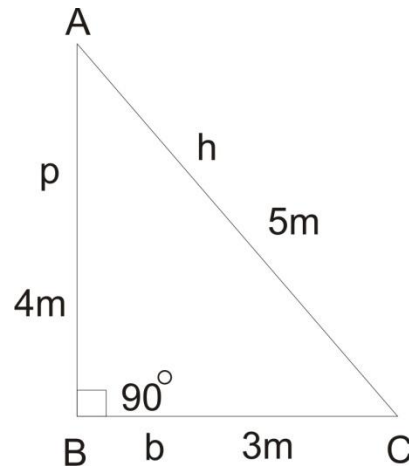
$$(AC)^2 = (AB)^2 + (BC)^2$$

or, $h^2 = p^2 + b^2$

If $AC = 5m$, $AB = 4m$ then

$$\begin{aligned}(BC)^2 &= (AC)^2 - (AB)^2 \\ &= 25 - 16 = 9\end{aligned}$$

$\therefore BC = 3m$



Rectangle : A rectangle is a plane,

Whose opposite sides are equal and

diagonals are equal. Each angle is

equal to 90° .

Here $AB = CD$; length $l = 4m$

$AD = BC$; breadth $b = 3m$

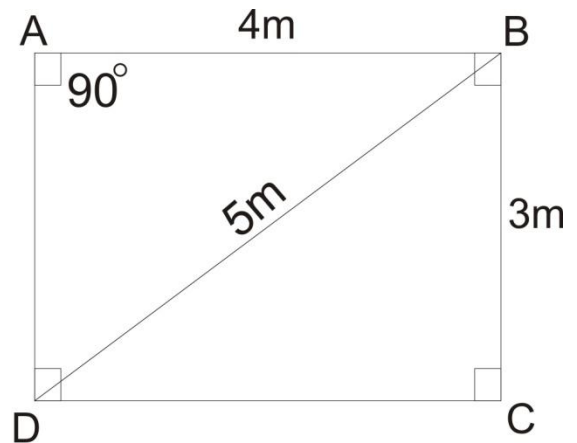
1. Perimeter of a rectangle = $2(\text{length} + \text{breadth})$

$$= 2(l + b)$$

$$= 2(4 + 3) = 14 \text{ m}$$

2. Area of rectangle = length x breadth = $l \times b = 4 \times 3$

$$= 12 \text{ m}^2$$



$$3. \text{ Length of a rectangle : } \frac{\text{area}}{\text{breadth}} = \frac{A}{b} = \frac{12}{3} = 4 \text{ m}$$

$$\text{or, } \left[\frac{\text{perimeter}}{2} - \text{breadth} \right] = \left(\frac{14}{2} - 3 \right) = 4 \text{ m}$$

$$\text{Breadth of a rectangle : } \frac{\text{area}}{\text{length}} = \frac{A}{l} = \frac{12}{4} = 3 \text{ m}$$

$$\text{or, } \left[\frac{\text{perimeter}}{2} - \text{length} \right] = \left(\frac{14}{2} - 4 \right) = 3 \text{ m}$$

$$4. \text{ Diagonal of rectangle : } \sqrt{(\text{length})^2 + (\text{breadth})^2}$$

$$= \sqrt{l^2 + b^2} = \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5 \text{ m}$$

Square : A square is a plane figure

Bounded by four equal sides having all its angle as right angles.

Here $AB = BC = CD = AD = 5 \text{ m} = a$ (Let)

$$1. \text{ Perimeter of square} = 4 \times \text{sides} = 4a$$

$$= 4 \times 5 = 20 \text{ m}$$

$$2. \text{ Area of a square} = (\text{sides})^2 = a^2 = (5)^2 = 25 \text{ sq. m}$$

$$3. \text{ Side of a square} = \sqrt{\text{area}} = \sqrt{25} = 5 \text{ m or, } \frac{\text{Perimeter}}{4} = \frac{20}{4} = 5 \text{ m}$$

$$4. \text{ Diagonal of a square} = \sqrt{2} \times \text{side} = \sqrt{2} a$$

$$= \sqrt{2} \times 5 = 5\sqrt{2} \text{ m}$$

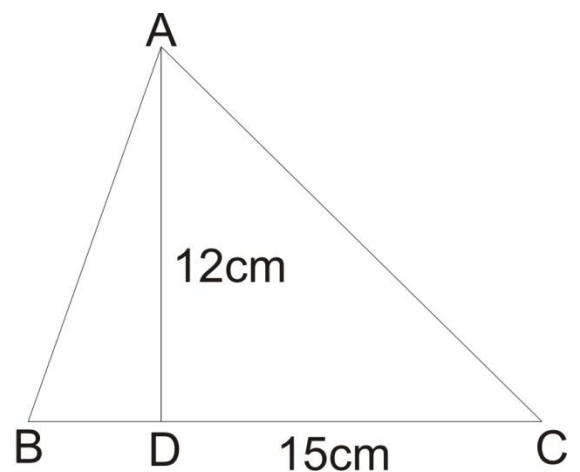
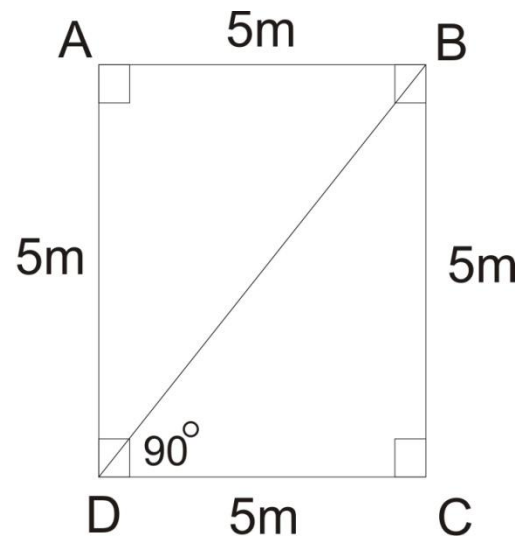
$$5. \text{ Side of a square} = \frac{\text{diagonal}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5 \text{ m}$$

Triangle :

$$1. \text{ Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 15 \times 12 = 90 \text{ sq. cm}$$

here $AD = 12 \text{ cm} = \text{height}$, $BC = 15 \text{ cm} = \text{base}$



2. Semi perimeter of a triangle

$$S = \frac{a+b+c}{2} = \frac{10+8+6}{2} = 12 \text{ cm}$$

here BC = a, AC = b, AB = c

3. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

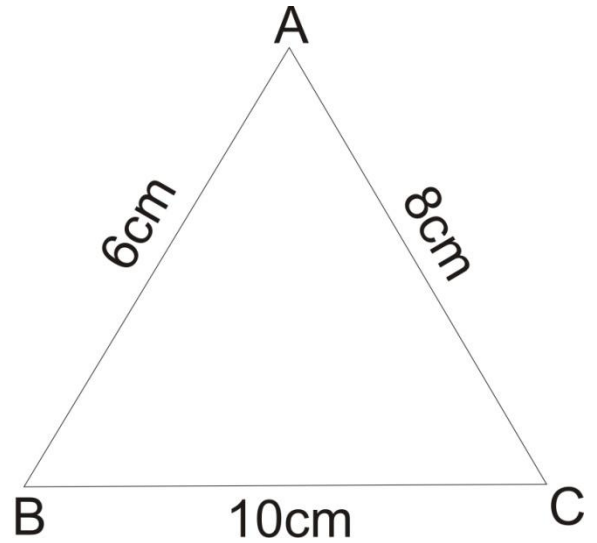
where a = 10cm, b = 8cm, c = 6cm, s = 12cm

$$= \sqrt{12(12-10)(12-8)(12-6)}$$

$$= \sqrt{12 \times 2 \times 4 \times 6} = 24 \text{ cm}^2$$

4. Perimeter of a triangle = 2s = (a + b + c)

$$= 10 + 8 + 6 = 24 \text{ cm}$$



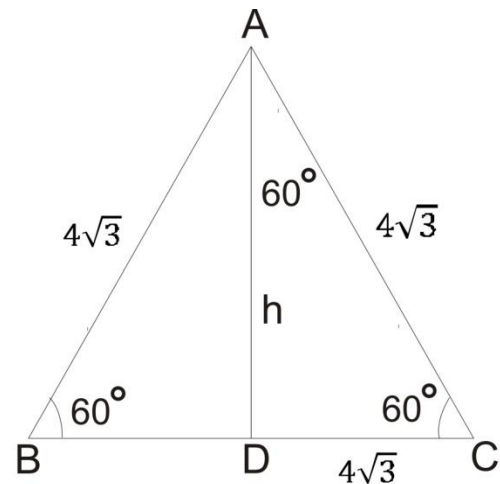
5. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3} \text{ cm}^2$$

6. Height of an equilateral triangle = $\frac{\sqrt{3}}{2} \times (\text{side})^2 = \frac{\sqrt{3}}{2} \times 4\sqrt{3}$

$$= 6 \text{ cm}$$



7. Perimeter of an equilateral triangle = 3 x (side)

$$= 3 \times 4\sqrt{3} = 12\sqrt{3} \text{ cm}$$

Quadrilateral :

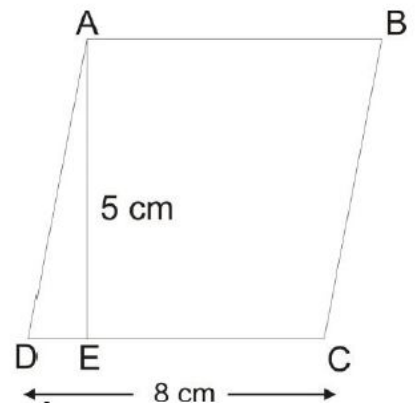
Parallelogram :

(i) Area of parallelogram = base x height

$$= b \times h$$

$$= 8 \times 5 = 40 \text{ sq.cm.}$$

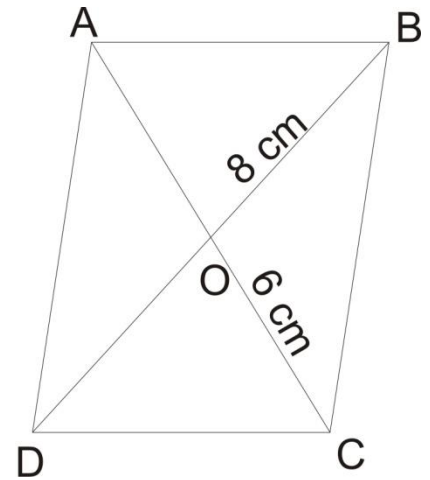
(ii) Perimeter of a parallelogram = 2(AB + BC)



$$= 2(8 + 5) = 26 \text{ cm}$$

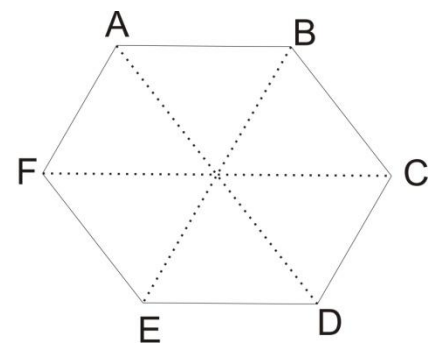
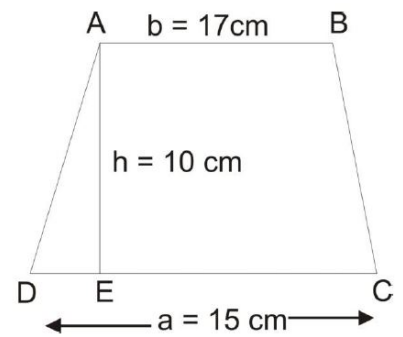
Rhombus :

- (i) Area of rhombus = $\frac{1}{2}$ x (product of diagonals)
- $$= \frac{1}{2} (d_1 \cdot d_2) = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$
- (ii) Perimeter of rhombus = 4 x side = 4a
- here AB = BC = CD = AD = 4a
- AC = d₁, BD = d₂



Trapezium :

- (i) Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x height
- $$= \frac{1}{2} \times (a + b) \times h$$
- $$= \frac{1}{2} \times (15 + 17) \times 10$$
- $$= \frac{1}{2} \times 32 \times 10 = 160 \text{ cm}^2$$

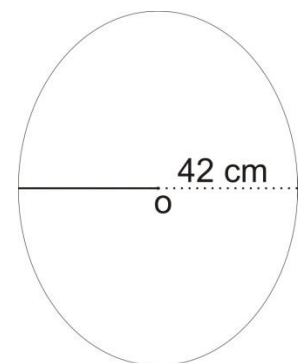


Regular Hexagon :

- (i) Area of a regular hexagon = $6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2$
- (ii) Perimeter of a regular hexagon = 6 x (side)

Circle :

- (i) Circumference of a circle = $\pi \times \text{diameter}$
- $$= \pi \times 2r = 2\pi r$$



$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

(ii) Radius of a circle = $\frac{\text{circumference}}{2\pi} = \frac{264 \times 7}{2 \times 22} = 42 \text{ cm}$

(iii) Area of a circle = $\pi \times r^2 = \frac{22}{7} \times 42^2 = \frac{22}{7} \times 42 \times 42 = 5544 \text{ cm}^2$

(iv) Radius of a circle = $\sqrt{\frac{\text{area}}{\pi}}$

$$= \sqrt{\frac{5544}{22}} \times 7 = \sqrt{1764} = 42 \text{ cm}$$

(v) Area of a semi circle = $\frac{1}{2} \pi r^2 = \frac{1}{8} \pi d^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 42^2 = 2772 \text{ cm}^2$$

(vi) Circumference of semi circle = $\frac{22}{7} \times 42 = 132 \text{ cm}$

(vii) Perimeter of semi circle = $(\pi r + 2r) = (\pi + 2) r = (\pi + 2) \frac{d}{2}$

(viii) Area of sector OAB = $\frac{x}{360} \times \pi r^2$

(x being the central angle)

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 3.5 \times 3.5 = 3.21 \text{ sq. m.}$$

(ix) Central angle by arc AB = $360^\circ \times \frac{\text{area of OAB}}{\text{area of circle}}$

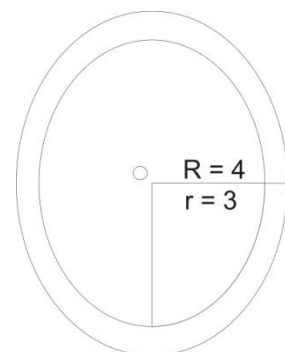
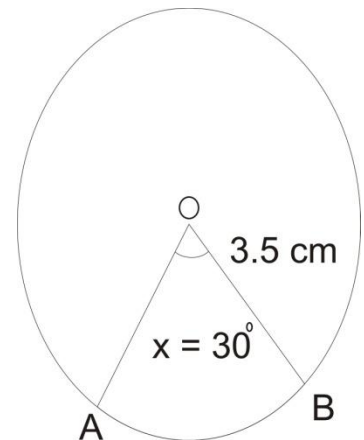
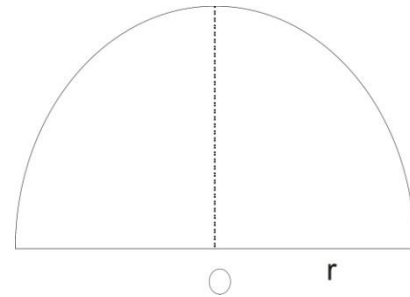
$$= 360^\circ \times \frac{3.21}{\frac{22}{7} \times 3.5 \times 3.5} = \frac{360 \times 321}{22 \times 35 \times 5} = 30^\circ (\text{approx})$$

(x) Radius of circle = $\sqrt{\frac{360^\circ}{\text{central angle by arc}} \times \frac{\text{area of OAB}}{\pi}}$

$$= \sqrt{\frac{360^\circ}{30^\circ} \times \frac{3.21}{\frac{22}{7}}} = \sqrt{\frac{134.82}{11}} = \sqrt{12.23} = 3.5 \text{ m.}$$

(xi) Area of ring

= difference of the area of two circle



$$\begin{aligned}
&= \pi R^2 - \pi r^2 = (R^2 - r^2) \\
&= \pi(R + r)(R - r) \\
&= (\text{sum of radius})(\text{diff. of radius}) \\
&= \frac{22}{7} \times (4 + 3)(4 - 3) = \frac{22}{7} \times 7 \times 1 \\
&= 22 \text{ sq. cm.}
\end{aligned}$$

Cuboid and Cube :

- (i) Total surface area of cuboid

$$= 2(lb + bh + hl) \text{ sq. unit}$$

Here l = length, b = breadth, h = height

$$= 2(12 \times 8 + 8 \times 6 + 6 \times 12)$$

$$= 2(96 + 48 + 72) = 2 \times 216 = 432 \text{ sq. cm.}$$

- (ii) Volume of a cuboid = (length \times breadth \times height) = lbh

$$= 12 \times 8 \times 6 = 576 \text{ cuboic cm}$$

- (iii) Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 8^2 + 6^2}$

$$= \sqrt{144 + 64 + 36} = \sqrt{244} = 2\sqrt{61} \text{ cm.}$$

- (iv) Length of cuboid = $\frac{\text{Volume}}{\text{Breadth} \times \text{Height}} = \frac{v}{b \times h}$

- (v) Breadth of cuboid = $\frac{\text{Volume}}{\text{Length} \times \text{Height}} = \frac{v}{l \times h}$

- (vi) Height of cuboid = $\frac{\text{Volume}}{\text{Length} \times \text{Breadth}} = \frac{v}{l \times b}$

- (vii) Volume of cube = (side)³

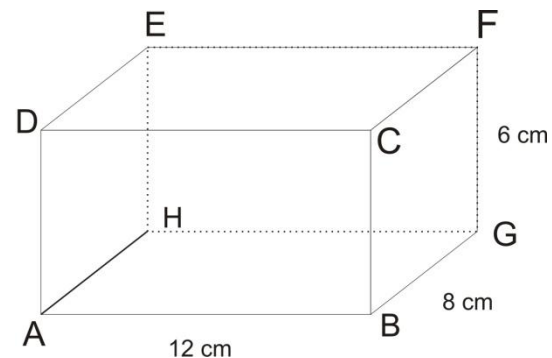
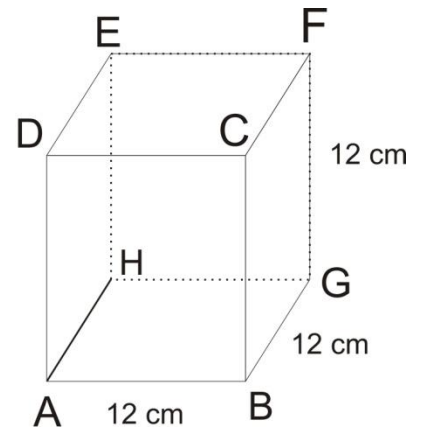
$$= 12^3$$

$$= 1728 \text{ cubic cm}$$

Cube : All sides are equal = 12 cm

- (viii) Sides of a cube = $\sqrt[3]{\text{Volume}}$

$$= \sqrt[3]{1728} = 12 \text{ cm}$$



- (ix) Diagonal of cube = $\sqrt{3} \times (\text{side}) = \sqrt{3} \times 12 = 12\sqrt{3}$ cm
 (x) Total surface area of a cube = $6 \times (\text{side})^2 = 6 \times 12^2 = 864$ sq.cm

Right Circular Cylinder :

- (i) Area of curved surface

$$= (\text{perimeter of base}) \times \text{height}$$

$$= 2\pi rh \text{ sq. unit}$$

$$= 2 \times \frac{22}{7} \times 7 \times 15 = 660 \text{ sq. cm}$$

- (ii) Total surface area = area of circular ends + curved surface area

$$= 2\pi r^2 + 2\pi rh = 2\pi r(r + h) \text{ sq. unit}$$

$$= 2 \times \frac{22}{7} \times 7(15 + 7)$$

$$= 2 \times 22 \times 22$$

$$= 968 \text{ sq. cm.}$$

- (iii) Volume = (area of base) x height

$$= (\pi r^2) \times h = \pi r^2 h$$

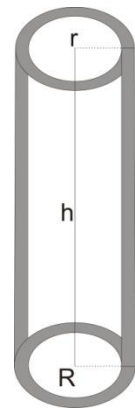
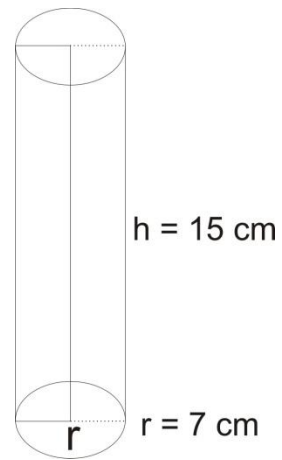
$$= \frac{22}{7} \times 7 \times 7 \times 15 = 2310 \text{ cubic cm.}$$

- (iv) Volume of a hollow cylinder = $\pi R^2 h - \pi r^2 h$

$$= \pi h(R^2 - r^2) = \pi h(R + r)(R - r)$$

$$= \pi \times \text{height} \times (\text{sum of radii})(\text{difference of radii})$$

Here R, r are outer and inner radii respectively and h is the height.

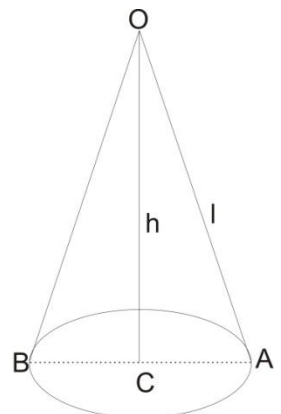


Cone :

- (i) In right angled ΔOAC , we have

$$l^2 = h^2 + r^2$$

(here $r = 35$ cm, $l = 37$ cm, $h = 12$ cm)



$$\text{Or, } l = \sqrt{h^2 + r^2}$$

$$h = \sqrt{l^2 - r^2}, \quad r = \sqrt{l^2 - h^2}$$

where l = slant height, h = height, r = radius of base

(ii) Curved surface area = $\frac{1}{2}$ x (perimeter of base) x slant height

$$= \frac{1}{2} \times 2\pi r \times l = \pi r l \text{ sq. unit}$$

$$= \frac{22}{7} \times 35 \times 37 = 4070 \text{ sq. cm}$$

(iii) Total surface area S = area of circular base + curved surface area

$$= (\pi r^2 + \pi r l) = \pi r(r + l) \text{ sq. unit}$$

$$= \frac{22}{7} \times 35(37 + 35) = 7920 \text{ sq. cm}$$

(iv) Volume of cone = $\frac{1}{3}$ (area of base) x height

$$= \frac{1}{3} (\pi r^2) \times h = \frac{1}{3} \pi r^2 h \text{ cubic unit}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12$$

$$= 15400 \text{ cubic cm}$$

Frustum of Cone :

(v) Volume of frustum = $\frac{1}{3} \pi h(R^2 + r^2 + Rr)$ cubic unit

(vi) Lateral surface = $\pi l(R + r)$

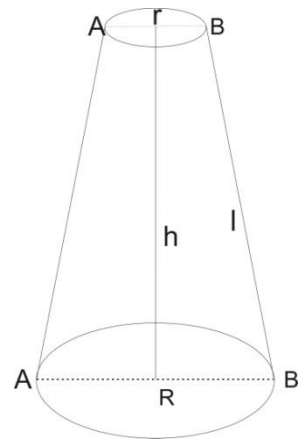
$$\text{where } l^2 = h^2 + (R - r)^2$$

(vii) Total surface area = $\pi[R^2 + r^2 + l(R + r)]$

R, r be the radius of base and top the frustum

$ABB'A'$ h and l be the vertical height and slant

height respectively.



Sphere :

(i) Surface area = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2 = 1386 \text{ sq. cm}$$

here, $d = 21 \text{ cm}$ $\therefore r = 10.5 \text{ cm}$

(ii) Radius of sphere = $\sqrt{\frac{\text{Surface area}}{4\pi}} = \sqrt{\frac{1386 \times 7}{4 \times 22}} = 10.5 \text{ cm}$

(iii) Diameter of sphere = $\sqrt{\frac{\text{Surface}}{4\pi}} = \sqrt{\frac{1386 \times 7}{22}} = 21 \text{ cm}$

(iv) Volume of sphere $V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{1}{6} \pi d^3$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \times 21 = 4831 \text{ cubic cm}$$

(v) Radius of sphere = $\sqrt{\frac{3}{4\pi} \times \text{Volume of sphere}}$

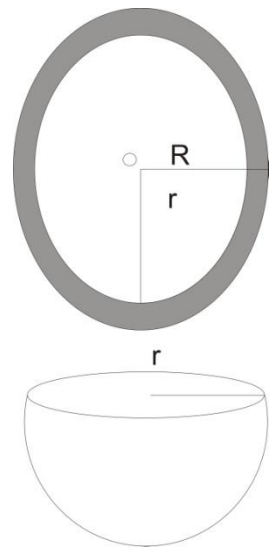
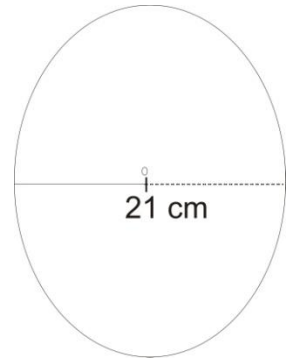
(vi) Diameter = $\sqrt[3]{\frac{6 \times V}{\pi}}$

(vii) Volume of spherical ring = $\frac{4}{3} \pi (R^3 - r^3)$

(viii) Curved surface of hemisphere = $2\pi r^2$

(ix) Volume of hemisphere = $\frac{2}{3} \pi r^3$

(x) Total surface area of hemisphere = $3\pi r^2$



Note : $V =$ volume, $A =$ area, $h =$ height, $b =$ base, breadth, $d =$ diameter, $R =$ outer radius, $r =$ inner radius, $\pi = \frac{22}{7} = 3.142$, $a =$ side.

Prism and Pyramid

Prism

1. **Solid :** Bodies which have three dimensions in space are called solid. For example, a block of wood.

A body, which has the three dimensions length, breadth and height, is a solid, whereas a rectangle with its two dimensions (length and breadth) is not a solid.

2. **Prism :** A prism is a solid, bounded by plane faces of which two opposite sides known as bases are parallel and congruent polygons.

3. **Base :** The congruent and parallel faces of a

prism are called its bases.

The other faces of a prism can be either oblique to the faces or perpendicular to them.

4. Right prism : A right prism is a prism in which lateral sides are rectangular or perpendicular to their bases.

5. Lateral faces : The side faces of a prism are called its lateral faces.

6. Lateral surface area : The area of all the lateral faces of a prism is called its lateral surface area.

Note : In a right prism having polygons of n sides as bases.

- (i) the number of vertices = $2n$
- (ii) the number of edges = $3n$
- (iii) the number of lateral faces = $(n + 1)$, and
- (iv) all the lateral faces are rectangular.

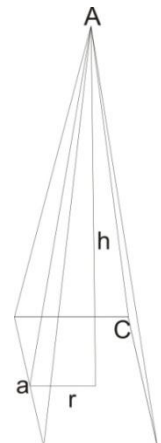
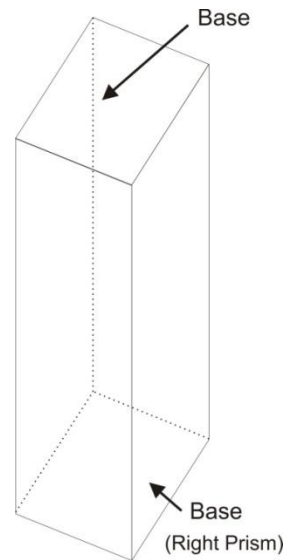
Formulae

- | | |
|-------|--|
| (i) | Volume of a right prism = (Area of its base) x height |
| (ii) | Lateral surface area of a right prism
=(perimeter of its base) x height |
| (iii) | Total surface area of a right prism
=(lateral surface area) + 2(area of the base) |

Pyramid

1. Pyramid : A solid of triangular lateral sides having a common vertex and plane rectilinear bases with equal sides is called pyramid.

2. Height of the pyramid : The length



of perpendicular drawn from the vertex
of a pyramid to its base is called the
height of the pyramid.

The side faces of pyramid form its lateral surface.

3. **Regular pyramid** : If the base of a pyramid is a regular figure i.e., a polygon with all sides equal and all angles equal, then it is called a regular pyramid.
4. **Right pyramid** : If the foot of the perpendicular from the vertex of a pyramid to its base is the centre of the base then it is called a right pyramid.
5. **Slant height of a regular right pyramid** : The slant height of a regular right pyramid is the length of the line segment joining the vertex to the mid-point of one of the sides of the base.
6. **Tetrahedron** : When the base of a right pyramid is a triangle, then it is called a tetrahedron.
7. **Regular tetrahedron** : A right pyramid with equilateral triangle as its base is called a regular tetrahedron.